Professional knowledge of future primary teachers - how can diagnostic competency be fostered?

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Abstract

The aim of this mini-course is to show an approach to develop prospective primary teachers’ diagnostic competency. Every teacher encounters errors in their students' mathematical work. However, the identification, understanding and appropriate response to those errors is not a straightforward process. It requires a competency that is a crucial part of mathematics teachers' professional competence. During the mini-course, the theoretical background of the promotion of diagnostic competency will be presented and participants will be encouraged to actively participate in an error analysis teaching sequence. Finally, the characteristics of the activity will be discussed with the participants, considering theoretical perspectives and the challenges of initial teacher education.

Key words: diagnostic competency, error analysis, mathematics education, initial teacher education.

Resumen

El objetivo de este mini-curso es dar a conocer una propuesta para desarrollar la competencia diagnóstica de estudiantes de pedagogía básica. Todos los profesores se encuentran con errores matemáticos en los trabajos de sus alumnos. Sin embargo, la identificación, comprensión y adecuada respuesta a los errores no es un proceso sencillo. Se requiere una competencia específica que es un componente crucial de la competencia profesional de los profesores de matemática. Durante el mini-curso, se presentarán las bases teóricas del desarrollo de la competencia de diagnóstico y se animará a los asistentes a participar activamente en una secuencia pedagógica sobre análisis de errores. Por último, las características de la actividad serán discutidas con los participantes, teniendo en cuenta las perspectivas teóricas y los desafíos de la formación inicial docente.

Palabras clave: competencia diagnóstica, análisis de errores, educación matemática, educación inicial del profesorado.

Introduction

Teaching mathematics for understanding is a key goal of educational reform in many countries that are shifting from a model of knowledge transmission towards a student-centered paradigm in which students' thinking is highlighted and taken as a starting point to build further mathematical knowledge. Such teaching is complex and so as to be effective in the primary education classroom, it requires teachers equipped with a specialized body of knowledge that has been described and researched in the past decades (Shulman, 1986; Ball, Thames & Phelps, 2008; Kaiser et al., 2014). Also, teachers' abilities to understand students' thinking is crucial, because they need to use this understanding as a foundation base to provide effective instructional strategies.

Errors are unavoidable in teaching-and-learning-processes, furthermore their educational potential has been recognized in the past decades, because errors serve as a window into students' mathematical thinking. Therefore, a key competence that future teachers need to develop during their university studies is their *diagnostic competence*, which constitutes a real challenge for teacher educators.

In this mini-course we will explore the theoretical background of this professional competence of primary mathematics teachers, a teaching sequence will be presented and experienced by participants and, finally, challenges considering teacher education will be discussed.

Professional knowledge of future primary teachers

The conceptualization of what constitutes the professional knowledge required by teachers started to change in 1986 with Lee Shulman's contribution. He suggested that subject-matter knowledge and pedagogical or curricular knowledge as separated domains were not enough for effective teaching. He pointed out that teachers also need what he called *pedagogical content knowledge* and defined it as including 'the ways of representing and formulating the subject that make it comprehensible to others' (Shulman, 1986, p. 9). In other words, it is a specialized knowledge of the subject needed by teachers to foster successful learning. It comprises, for instance, a wide variety of examples, representations and explanations that would allow students' learning, knowledge about which aspects of a specific content make it easier or more difficult for students' to grasp, the most common errors and difficulties for each topic and a range of strategies that may help students overcome their difficulties. The IEA study "Teacher Education and Development Study in Mathematics" (TEDS-M) developed a theoretical framework for its international comparative study referring to this seminal work by Shulman (1986) and distinguished mathematical content knowledge (MCK), mathematics pedagogical content knowledge (MPCK) and general pedagogical knowledge (GPK). These domains can be empirically distinguished, but are strongly related (for a short overview see Kaiser et al., 2014). In particular, MPCK is described as consisting of mathematical curricular knowledge, knowledge required to plan or design mathematics teaching and learning strategies (pre-active) and the knowledge required to effectively implement those strategies and interact in the classroom (inter-active) (Tatto et al., 2008). However, until now, no consensus exists about the conceptualization of the professional knowledge of teachers, already many studies depart from the seminal work of Shulman (1986). Especially, the description of MPCK varies strongly as Depaepe, Verschaffel and Kelchtermans (2013) point out.

Loewenberg Ball and colleagues elaborated Shulman's construct of pedagogical content knowledge especially focusing on primary teachers and clarified further how teachers are expected to know the contents they teach (Ball, Thames & Phelps, 2008). Their analyses yielded that 'the mathematical demands of teaching are substantial. The mathematical knowledge needed for teaching is not less than that needed by other adults. In fact, knowledge for teaching must be detailed in ways unnecessary for everyday functioning.' (Ball et al., 2008, p. 396). In view of this, they created the concept of *Mathematical Knowledge for Teaching* (MKT) and described it by organizing its subcomponents in two areas: Subject Matter Knowledge and Pedagogical Content Knowledge (PCK). Within PCK, they included three domains, namely knowledge of content and curriculum, knowledge of content and students (KCS) and knowledge of content and teaching (KCT). Knowledge of content and students includes the ability to anticipate which aspects of a particular content can be confusing for students or the type of reasoning that children could follow, the ability to interpret and understand the arguments sometimes incomplete and expressed in everyday language of young students and knowledge about the most common errors that may arise during the learning of certain mathematical content. Similarly, teachers put into practice their knowledge of content and teaching when, for example, during a whole-class discussion they must decide whether they deepen or not in the contribution of a student, if they stop to better clarify an issue, if they make a question or give a particular task to enhance or put into conflict the reasoning of a student (Hill, Ball & Schilling, 2008; Ball et al., 2008).

These distinctions acknowledge the relevance of understanding students thinking. For instance, how could teachers decide whether the contribution of a student is worth following in whole-class discussion if they do not understand the reasoning behind that student's claim? Or more generally, how could they foster the development of students' mathematical thinking if they do not comprehend the starting point of their students' mathematical understandings? It is clear that interpreting, analyzing and understanding students thinking provide teachers with useful information to make instructional decisions before, during and after each lesson. Therefore, it is of special interest the competences of teachers related to understanding students mathematical thinking, because they facilitate teaching practices aimed at providing and attending to the educational needs of all learners (Cooper, 2009; Empson, 2003).

Diagnostic competence: understanding learners' thinking as a key professional competence

Although the concept of *diagnostic competence* might have some clinical or medical connotations in some languages, it has also been used in reference to 'a teacher's competence to analyse and understand student thinking and learning processes without immediately grading them' (Prediger, 2010, p. 76). This is in line with what is being argued worldwide. If school systems are aimed at promoting the development of children's mathematical literacy (OECD, 2012) or at promoting learning mathematics for understanding (NCTM, 2000; Van de Walle, Lovin, Karp & Bay-Williams, 2014), they need teachers equipped with the competencies to identify and understand what students know, what they still need to learn and what they have misunderstood, with the abilities to make ongoing analyses of children's learning and new understandings and to make instructional decisions aimed at supporting and challenging them in the learning process (NCTM, 2000).

Similarly, Prediger (2010) suggests that teachers benefit from their diagnostic competence, because in a learner-centered paradigm, effective teaching should consider students current level of mathematical understandings as the foundation for building further knowledge. In other words, teachers support in the learning process should start from children's ideas and to do so, they require the ability to analyze and understand a wide variety of students' thinking that might be communicated incompletely. Unless teachers are very clear about where students' knowledge and skills need further support to improve, it is difficult to begin discerning what pedagogical resources might help students learn.

According to Prediger (2010) diagnostic competence is made-up of four elements. First, teachers need to be interested and curious about student thinking. Second, teachers need to assume an 'interpretative attitude of understanding from an inner perspective' (p. 77) that allows them to go further and not only identify the correctness of students' work, but also to understand the underlying reasoning of student thinking. Additionally, teachers need theoretical knowledge about mathematics learning as a background to analyze and understand student thinking. Finally, Prediger suggests that this knowledge needs to be complemented with mathematical knowledge specific to each mathematical concept that would allow teachers to analyze and understand student thinking according to the different meanings of each particular concept.

This suggests that diagnostic competence should not be viewed as related to a single dimension of *mathematical knowledge for teaching* as described by Ball and her colleagues (2008) nor only to *pedagogical content knowledge* as conceptualized by Shulman (1986). It implies a more 'integrated understanding of mathematical knowledge for teaching' (Prediger, 2010, p. 79). Thus, teacher educators must provide future teachers with well-thought and complex opportunities to learn, where the elements distinguished above interact.

Mathematical errors: a window into students' mathematical thinking

Students' mathematical errors, considered as 'systematic, persistent and pervasive mistakes' (Brodie, 2014, p. 223) that are not easily identified and corrected by learners themselves, are a good opportunity for teachers to look into children's mathematical understanding.

Persistent and systematic errors are explained by constructivism as the result of erroneous conceptualizations or misconceptions which, in turn, are cognitive structures built by students according to their previous knowledge and experiences or by overgeneralizing knowledge from other domains (Smith, diSessa & Roschelle, 1993). As a consequence, errors make sense for the student, because there is an underlying (erroneous) reasoning explaining the error, frequently connected to some other correct knowledge (Brodie, 2014).

According to Brodie (2014), as a result of this connection to correct conceptual structures in other domains, errors are difficult to eradicate, because complex cognitive restructuring needs to take place. This involves a process in which learners must recognize that what until now makes perfect sense to them is not correct and teachers need to be able to select strategies that are more likely to help students reorganize their understandings.

Brodie (2014, p. 224) synthesizes the characteristics of errors stating that

> 'errors are reasonable and show reasoning among learners; they are a normal and necessary part of learning mathematics; and learner errors give teachers access to learners' current thinking about and ways of doing mathematics and access to possibilities for future growth in their mathematical thinking and practices.'

Because errors provide teachers with an opportunity to understand student thinking and hence, design and deliver appropriate learning experiences, errors need to be considered and addressed during teaching (Smith et al., 1993; Brodie, 2014). In fact, there is strong evidence showing that doing so is more effective for learning than trying to avoid or ignore the occurrence of errors in the classroom (see Keith & Frese, 2008).

Fostering diagnostic competence in teacher education courses

Considering the relevance of teachers' understandings of student thinking, it is evident that developing diagnostic competence needs to be a crucial aim during pre-service teacher education (Bartell, Webel, Bowen & Dyson, 2013; Cooper, 2009). Although teaching experience is key in developing this kind of competences, according to Cooper (2009) future teachers would certainly benefit from guided experiences and knowledge that may constitute the basis for the development of this competence along their professional career. The question that follows is how this competence can be fostered.

Jacobs & Philipp (2004) propose that both written work from students and videos showing children's work should be used in teacher education to promote analysis and discussion about student thinking and hence, facilitate the development of knowledge related to mathematics, teaching and learning. They also make the point that the value of bringing those work samples into teacher education courses is not in the children's work themselves, but in the discussions that may arise when teacher educators provide with well-chosen questions and prospective teachers engage in fruitful analyses and conversations about mathematics teaching and learning.

In a subsequent work, Jacobs, Lamb and Philipp (2010) go on to describe some prompts that may help teacher educators to encourage those productive discussions. They organize the discussion prompts into their *noticing of children's mathematical thinking* framework, which comprises three fundamental skills: attending to children's strategies, interpreting their understandings and deciding how to respond on the basis of this analysis. They make the significant claim that these three skills need to be considered in an integrated way, not independently or sequentially.

For each of these skills they give some guidance on how they may be addressed in teacher education courses. For instance, they suggest that in attending to children's strategies is not only important the identification of relevant elements in a complex classroom environment, but also the recognition of what is mathematically significant in students' explanations, and teacher educators need to provide directed support so future teachers can start the development of this skill in an appropriate way. Regarding the ability to interpret children's understandings, the authors highlight the need to focus student teachers efforts on analyzing learners' understandings specifically and not in the discussion on more general issues. Their difficulties to interpret children's work may be the result of a deficit on the mathematical knowledge needed to understand their strategies and make the appropriate connections to mathematical concepts.

Lastly, Jacobs, Lamb and Philipp (2010) state that the ability to give an appropriate pedagogical response, needs to be based on children's understandings. The type of responses that student teachers may suggest can vary widely, including testing teacher's analysis of children's understandings, exploring deeper students' solutions and their underlying reasoning or proposing a new problem. In any case, to be productive, they should consider students' understandings.

Similarly, Peng and Luo (2009) propose a framework that describes mathematics teacher knowledge related to error analysis. They identified four types of error analysis: (i) *identify* or detect the presence of a mathematical error, that could be related to the ability to attend to children's understanding from the 'noticing' framework; (ii), *interpret* the underlying rationality of a student mathematical error and (iii) *evaluate* the level of performance of a student, according to what the mathematical error shows, that is clearly connected with the interpreting skill proposed by Jacobs and her research team; and (iv) *remediate* the mathematical misconception by presenting a targeted teaching strategy to eliminate mathematical error, that can also be linked to the third skill of 'noticing', namely the ability to decide how to respond based on children's understandings.

Accordingly, McGuire (2013) designed an error analysis problem structure that included three separate but related levels: identify students' error pattern or misconception, 'think like a student' or answer similar problems using the same error pattern and describe remediation strategies.

In this work, we consider an integration of these models that can be summarized in the diagram in Figure 1.

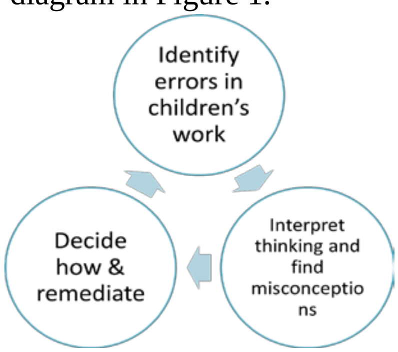

Figure 1. Error diagnosis cycle.

This model proposes that prospective teachers should start by attending to children's work and identifying the systematic errors when they appear, then they should interpret the underlying thinking of students and locate where misconceptions are placed. With this in mind, future teachers need to activate their knowledge related to mathematics, teaching and learning to decide which aspects of children's knowledge need further improvement and design carefully thought and targeted instructional strategies.

Error analysis teaching sequence structure

A four-session-teaching sequence was designed with the aim of fostering the development of prospective primary teachers' diagnostic competence. During the four 90 minute sessions, prospective teachers are expected to engage in individual analysis and in small and large group discussions of children's work, coming both from written samples and from video clips taken from mathematical classrooms. It is intended that the prospective teachers work through the error analysis cycle several times, based on different samples of primary students' mathematical work and that they will communicate their proposals and evaluate those of their classmates.

The first session is aimed at sensitizing prospective teachers' interest on error analysis and understanding of students thinking as a way into providing effective pedagogical strategies that promote children's learning of mathematics. The error analysis cycle will be presented, explained and applied to study the work of primary students.

The goals of the second session are to highlight the opportunity that error analysis give to look into children's understandings and to develop prospective teachers' abilities of identifying and comprehending students' mathematical reasoning and misconceptions. This session focuses especially on the first and second stage of the error analysis cycle (i.e. identifying and interpreting) and also guides future teachers to acknowledge that primary school students think differently than adults do.

The third session focuses primarily on the development of prospective teachers' ability to provide effective pedagogical responses to students' errors. In order to do this, they are expected to use their previous knowledge and information provided during the session to analyze teachers' responses to students' errors in classroom settings and to design appropriate instructional strategies to address those errors. In this session future teachers will be working through the whole error analysis cycle.

The last session is intended to apply knowledge and abilities developed in previous sessions. Prospective teachers are expected to analyze the work from a student. They will identify and interpret written work from a student to comprehend the underlying reasoning of the learner errors and design an effective response. In addition, they will communicate their analyses to other groups and evaluate how others have analyzed the students' errors. In this way, they will have access to a wider variety of errors and of points of view.

Participation in this mini-course

During this mini-course, participants will be encouraged to actively participate in selected activities from the second session. Particularly, they will be asked to analyze and discuss four pieces of students' work on fractions, identify the error underlying each of them, interpret students' thinking and find their misconceptions and solve similar tasks using the students' reasoning. After discussing those analyses, participants will watch a video showing a student working on one of the previous cases. This will be followed by a pair discussion of the video, aid by some given prompts, focusing on the identification, interpretation and understanding of the misconception. In a whole-group discussion, participants will also be encouraged to discuss adequate teaching alternatives, which might be used in further teaching practice.

To conclude the description activities of the mini-course, it is expected that the participants engage in a discussion about the characteristics of the activity, considering theoretical perspectives and the challenges of pre-service teacher education.

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