Mathematics Knowledge for Teaching at the Secondary Levels: Methods and Evidence from the TEDS-M Study

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Summary

This presentation describes the results of the Teacher Education and Development Study in Mathematics (TEDSM) a collaborative study of the mathematics preparation of future primary and secondary teachers. The study explored the question of whether what future teachers learn in pre-service teacher education leads to more effective knowledge of mathematics and mathematics for teaching. The methods and results of TEDSM are examined in light of their contribution to a prospective international study of novice mathematics teachers known as The First Five Years of Mathematics Teaching or FIRSTMATH.

Keywords: mathematics education, teacher education, professional development, international comparisons

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The latest TIMSS 2011 results measuring mathematics achievement of 8th grade students shows twelve education systems (including four US and one Canadian state systems) scoring very high including Korea, Singapore, Chinese Taipei, Hong Kong, Japan, Massachusetts (in the USA), Minnesota (in the USA), the Russian Federation, the USA, North Carolina (USA), Quebec, and Indiana (USA) (with average scores ranging from 613 to 522; the US scored 509). TIMSS provides "benchmarks" to give meaning to the average scores obtained in the assessment; the cut scores for the different levels are 625 indicating "advanced" 550 indicating "high", 475 indicating "intermediate", and 400 indicating "low".

For instance eleven of the twelve systems described above had a higher percentage of students performing at the advanced level than the US system as a whole, indeed given the average score for the U.S., 8th graders on average perform between the intermediate and the high level, with relatively few reaching the advanced level. The description of the advanced level benchmark is included below (the other performance levels are described in the NCES TIMSS 2011 report):

At the advanced level, "[s]tudents can reason with information, draw conclusions, make generalizations, and solve linear equations. Students can solve a variety of fraction, proportion, and percent problems and justify their conclusions. Students can express generalizations algebraically and model situations. They can solve a variety of problems involving equations, formulas, and functions. Students can reason with geometric figures to solve problems. Students can reason with data from several sources or unfamiliar representations to solve multi-step problems" (Provasnik, S., Kastberg, D., Ferraro, D., Lemanski, N., Roey, S., and Jenkins, F., 2012, p.19).

This level of performance reflects the knowledge expected of secondary school graduates in the US as stated in curricular materials and implemented in schools (e.g., TIMSS 2011 reports that most US students are taught the major curricular topics of number 99% (versus 100% in Korea, and 99% in both C. Taipei and Singapore), algebra (86% in US, versus 97% in Chinese Taipei, 91% in Korea, and 94% in Singapore), geometry 87 in US (versus 92 in Korea, 84% in Chinese Taipei, and 75% in Singapore), and data and chance 91% in US (versus 81 Korea, 4% in Chinese Taipei, and 83% in Singapore)). Thus a substantial amount of time is dedicated to teaching mathematics in the US 157 hours per year surpassing Korea (137) and Singapore (138) but slightly lower than Chinese Taipei (166). Yet while the advanced level of knowledge is easily reached by a large proportion of pupils in the 11 systems listed above (and in particular in Korea (613), Singapore (611), and Chinese Taipei (609) all above one standard deviation from the international mean and from the US score (509)), US students find it difficult to attain this level of mathematics knowledge at the end of middle school. A more in depth look at the TIMSS data shows that the lower scores tend to be concentrated in poorer schools and among students of minority background.

But what do we know about the specific preparation of future U.S. secondary teachers? TIMSS 2011 found that while almost all of the eighth grade students were taught mathematics by teachers with postgraduate university degrees (62% had a doctorate, master's, or other postgraduate degree or diploma), or a bachelor's degree college degree (38%) not all had a mathematics degree or a mathematics education major. Twenty eighty percent of eighth grade students were taught mathematics by teachers who had a major in mathematics and in mathematics education with this group reporting the highest score (524); 25% were taught by teachers who had a major in mathematics education but not in mathematics (reporting a score of 510), while 15% were taught by teachers who had a major in mathematics but not in mathematics education (reporting the lowest score of 497). A large proportion of students (31%) were taught by teachers with other majors (Mullis, Martin, Foy and Arora, 2012). The TIMSS studies provide a general view in terms of degrees attained but fall short of describing what teachers actually know, the TEDS-M study provides this key information.

The TEDS-M Study

TEDS-M's primary purpose was to gather empirical evidence about mathematics teacher preparation for primary and lower secondary grades. The data included assessments of future teachers' knowledge and was collected via surveys during 2008-2009 from national representative sample of institutions, teacher educators and future teachers who were in their last year of their teacher preparation (see Tatto et al. 2012, or Tatto, 2013 for more detail). This article reports on the secondary teacher education findings, the primary teacher education findings have been reported elsewhere.

The questions that guided the TEDS-M study are:

(1) What is the level and depth of the mathematics and related teaching knowledge attained by prospective secondary teachers expected to enable them to teach the kind of demanding mathematics curriculum currently found in the higher achieving countries (and required by U.S. on-going standards-based reform)? Is this knowledge similar or different across countries?

(2) What are the learning opportunities available to prospective mathematics teachers at the secondary level and how are these structured? What is the content taught and what are the implementation processes of teacher education programs?

(3) What are the policies that support secondary teachers' achieved level and depth of mathematics and related teaching knowledge? How do these policies influence the structure of teachers' opportunities to learn mathematics at national and institutional levels, and how in turn do these contribute to the knowledge attained by these future mathematics teachers?

The TEDS-M study can be seen as a comparative investigation on how teachers are expected to acquire the knowledge conceived as needed to teach mathematics. While the primary purpose of TEDS-M was to investigate the mathematics knowledge for teaching as a function of the structure and content of pre-service teacher education, the data collected in the study across a number of countries also helps explain the degree to which teacher education policy is achieving the goal of preparing knowledgeable mathematics teachers for secondary teaching. This article provides a report of the findings for future secondary teachers and their programs in Botswana, Chile, Chinese Taipei, Germany, Malaysia, Oman, Philippines, Poland, the Russian Federation, Singapore, Switzerland (German-speaking cantons only), Thailand, and the United States (public institutions only).[1]

Methods[2]

Future Teachers, Teacher Education Programs and Routes Definitions

Defining what is meant by a future teacher and by a teacher education program were important first tasks for this study. For the purposes of TEDS-M a future teacher was defined as "a person enrolled in a teacher preparation program that is explicitly intended to prepare teachers qualified to teach mathematics in any of the grades at primary or lower secondary school level" (Tatto, 2013).

[1] In the collaborative tradition of IEA, the countries invite themselves to participate in IEA studies. For TEDS-M a total of 15,163 future primary teachers were surveyed in 451 institutions and 9,389 future secondary teachers were surveyed in 339 institutions in 16 countries participated in the TEDS-M study (see Tatto et al., 2012, for the TEDS-M final report). Other countries participated in the study (Canada, Georgia, and Norway) but the data collected did not meet the coverage requirement to ascertain representativeness. They are therefore not included in this article.

[2] A detailed description of the TEDS-M methods is in Tatto, 2013.

The program and the route were two concepts used to define teacher education. A program was defined as a specific pathway that exists within an institution that requires students to undertake a set of subjects and experiences, and leads to the award of a common credential or credentials on completion [..] a route is a set of teacher education programs available in a given country [and] share a number of common features that distinguish them from programs in other routes" (Tatto, 2013). TEDS-M identified two major routes:

"Concurrent routes consisting of a single [teacher education] program that includes studies in the subjects future teachers will be teaching (academic studies), studies of pedagogy and education (professional studies) and practical experience in the classroom; and consecutive routes consisting of a first phase for academic studies (leading to a degree or diploma), followed by a second phase of professional studies and practical experience (leading to a separate credential / qualification); the first and second phases need not have been completed in the same institution; and no route can be considered consecutive if the institution or the government authorities do not award a degree, diploma or official certificate at the end of the first phase" (for more detail see Tatto, 2013).

Sampling

The international sampling plan used a stratified multi-stage probability sampling design. The programs to study were randomly selected from a national list of teacher education programs, and future teachers were randomly selected from a list of in-scope future teachers for each of the randomly selected teacher preparation institutions. In smaller countries, all teacher preparation institutions were selected to participate in TEDS-M, and in some countries, all eligible future teachers in the sampled institutions were surveyed. While the samples are of unequal sizes these should be seen as representative of national systems of teacher education in the countries.[3]

Instruments[4]

The data reported in this article comes from the survey of teacher education programs and from the future teacher survey. The survey of teacher education programs consisted of questions asking information about the organization and content of the programs included in the study. The future teacher survey consisted of questions asking about background characteristics and opportunities to learn, and an assessment of mathematics knowledge for teaching which measured mathematics content knowledge and mathematics pedagogy content knowledge. The instruments were rigorously developed, and translated from the English to the local languages and back translated to confirm accuracy and consistency. Further details on the methods and design of the study can be found in the TEDS-M Conceptual Framework and in the Technical Report (Tatto et al., 2008, and Tatto, 2013). The content and reliability of the scores and scales is described below.

[3] The minimum sample size was set at 50 institutions per level; and an effective sample size of 400 future teachers per level in a given country. "Effective sample size" means that the sample design must be as efficient (i.e., precise) as a simple random sample of 400 teachers from a (hypothetical) list of all eligible future teachers.

[4] More details on the test and the study in general can be found in Tatto et al. 2008; and in Tatto, 2013.

Program Measures

Program characteristics and structure. A survey of teacher preparation institutions was conducted to collect data on institutional program characteristics and structure.

Opportunities to learn (OTL). A number of indices were included to allow exploration of the opportunities to learn that future mathematics teachers have across countries such as counts of topics studied in tertiary level mathematics, and in school level mathematics: (a) geometry taught at the tertiary level, which included topics such as foundations of geometry or axiomatic geometry, analytic/coordinate geometry, non-Euclidean geometry, and differential geometry; (b) upper school level mathematics, including functions, relations, equations, data representation, probability, statistics, calculus, and validation, structuring, and abstracting. For the mathematics topic indicators (or counts of courses taken), Fit Indices provided evidence that the groupings of the courses, based on logical organization as judged by experts, make sense given the data reported by future secondary teachers (for tertiary level mathematics CFI .969, TLI .986, RMSEA .032, and for school level mathematics CFI .892, TLI .846, RMSEA .085).[5]

All the other OTL indices are based on a 4-point scale (e.g., expressing frequency such as "never" to "often") and include topics such as mathematics education/pedagogy; education /pedagogy; accommodations to classroom diversity and reflections on practice; from school experience and the practicum; in a coherent teacher education program (e.g., whether each of the courses was clearly designed to prepare future teachers to meet a common set of explicit standard expectations for beginning teachers, whether there were clear links between most of the courses in the teacher education program and practicum experiences, and similar). Based on a series of confirmatory factor analyses, OTL indices were scaled using the Rasch model and are based on a score scale where 10 is located at the neutral position. The reliabilities for the opportunity to learn indices ranged from .83 to .97.[6]

Program philosophy. Program philosophy was defined as the sum of beliefs expressed by future teachers at the moment close to exiting their program and measured using 6-point rating scales (e.g., "strongly agree to strongly disagree") in five different areas; two of these scales are relevant to the current exploration. The "nature of mathematics scales" explored how future teachers perceive mathematics as a subject (e.g., mathematics as formal, structural, procedural, or applied), while the "learning mathematics scales" explored ideas about the

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[5] Comparative Fit Index (CFI): The CFI depends in large part on the average size of the correlations in the data. If the average correlation between variables is not high, then the CFI will not be very high. An acceptable model is indicated by a CFI larger than .93, but .85 is acceptable (Bollen 1989).The Tucker Lewis index (TLI) is relatively independent of sample size (Marsh, Balla, and McDonald 1988). Values over .90 or.95 are considered acceptable (e.g., Hu and Bentler 1999). Root Mean Square Error of Approximation (RMSEA): Another test of model fit, good models are considered to have a RMSEA of .05 or less. Models whose RMSEA is .1 or more have a poor fit.

[6] The reliabilities for the OTL and beliefs scales are unweighted and were estimated using jMetrik 2.1 (Meyer, 2011). The reliability estimates are based on the congeneric measurement model, which allows each item to load on the common factor at different levels and allows item error variances to vary freely (each item can be measured with a different level of precision). This is the most flexible measurement model and most appropriate for measures with few items. Reliabilities tend to be high if there is a lot of variation in the sample relative to the size of the standard error. The reliability will be low if one of the following occurs: (a) There is a small standard deviation in the sample or (b) there is a large standard error (e.g., the test was too easy for a particular sample).

appropriateness of particular instructional activities, questions about students' cognition processes, and questions about the purposes of mathematics as a school subject. Belief items were scaled using the Rasch model and are based on a score of 10 located at the neutral position. The reliability for the beliefs scales was for the "mathematics as a set of rules and procedures" scale .93; and for "learning mathematics through active involvement".92.

Program's wealth. Programs' socioeconomic status or wealth is an aggregated scale created by measures of future teachers' home possessions including number of books at home, and parents' levels of education.[7]

Future Teacher Measures

Background and prior attainment. The Future Teacher Questionnaire included questions about the background of respondents; specifically information about individuals' socioeconomic status or SES, age, gender, and prior attainment.

Assessing Knowledge for Teaching: Mathematics Content Knowledge and Mathematics Pedagogy Knowledge. The assessment of Mathematics Content Knowledge [MCK] measured four domains: number and operations, algebra and functions, geometry and measurement, and data and chance. The assessment framework for mathematics content followed closely that used in the Trends in Mathematics and Science Studies [TIMSS] (see Mullis, Martin, and Foy 2008; Garden, Lie, Robitaille, Angell, Martin, Mullis, Foy and Arora 2006). The test for Mathematical Pedagogical Content Knowledge [MPCK] was developed by the TEDS-M team and measured three domains: curricular knowledge, knowledge of planning for teaching, and knowledge of enacting teaching. Three blocks of items were assembled for the secondary test, each with 12 – 15 questions. Each future teacher received a booklet with two of the blocks of items about knowledge for teaching mathematics. The test was designed to take up to 60 minutes to answer under a controlled administration. To sample all the domains the study used a "matrix sampling" design for the assessments (Mazzeo, Lazer, and Zieky, 2006).

To obtain comparable estimates of performance, item response theory (IRT) was used. Item response theory allows estimates of performance to be obtained on the same scale even when the set of items taken by each individual is different (see, e.g., De Ayala, 2009). The first step in the process for forming the reporting score scales was to calibrate the test items and then evaluate the results to determine if the data were well fit by the IRT models. Items with poor fit were reviewed or removed from the score computation; the final sets of items were calibrated again using weights so that each country contributed equally to the calibration (Wu, Adams, Wilson, and Haldane 2007). The final calibration results were used to estimate the location of the examinees on a common IRT scale and were then transformed so that the international mean for the calibration sample on each of the MCK and MPCK scales was 500 and the international standard deviation was 100. For the international sample, the reliability was higher for the longer mathematics content knowledge assessment than for the shorter mathematics pedagogical content knowledge assessment (.91 and .72 respectively). To give concrete meaning to the

[7] Using principal components analysis, a scale was created to obtain a proxy measure of socioeconomic status, by averaging the possessions in the parents or guardians home variables (such as number of books at home, father's highest level of education and mother's highest level of education). Its aggregate within a program constitutes the variable program's wealth.

assessment results as reported in score scales the study team developed anchor points with corresponding descriptions (see Tatto, 2011).

Describing Secondary Teacher Education Programs and their Students

Program Characteristics and Structure

Table 1 shows the variation in the provision of teacher education for future secondary teachers. All the participating countries provide teacher education at the tertiary level to the majority of their future secondary teachers with programs located in universities or institutions of higher education and typically with periods of practice in collaboration with schools. Most countries provide a common curriculum for their future secondary teachers who are expected to teach up to grade 11 and above. In other countries however more than one program exists and teacher education is more specialized according to the lower and the upper levels of secondary schooling (e.g., preparing teachers to teach up to grade 10, and to teach up to grade 11 and above) this is the case of Botswana, Germany, Poland, Singapore, and the USA. Most future secondary teachers are expected to become mathematics specialists with the exception of Chile where they are prepared as “generalists”, and in Switzerland where they are considered as “generalists with some specialization”. In most countries teacher education occurs in concurrent programs (such as in Botswana, Chile, Chinese Taipei, Malaysia, the Philippines, Poland, the Russian Federation, and Switzerland), but some countries also have consecutive programs (as in Oman, Singapore, Thailand, and the USA), and in some there is a combination of both (as in Germany). The duration of the program varies (with a range of 3 to 6.5 years) depending on whether the program is concurrent or consecutive and the extent of the practicum experience.

For the next sections findings are presented in two different tables according to the number of teacher education programs sampled in each participating country. Table 5 shows the means and standard deviations for countries with larger samples of teacher education programs and Table 6 for countries with smaller samples.

Opportunities to Learn

Future secondary teachers are given exposure to university and school level mathematics topics, especially in Poland, Russia and Thailand (in the larger countries) and in Chinese Taipei and Oman (in the smaller countries) with means close to or above 3.5 out of 4 topics (see Tables 5 and 6). Chile shows the lowest exposure to both areas of mathematics knowledge followed by Botswana and Singapore (whose future secondary teachers enter teacher education with high levels of mathematics knowledge already thus the noted emphasis on the mathematics of the school curriculum) in university level mathematics, all with means lower than 2. Overall, programs place more emphasis in the areas of school level mathematics (with means close or above 3). The frequency with which future secondary teachers engaged in reading research on teaching and mathematics was particularly low in Germany, Switzerland, and Poland. Program coherence (understood as consistency within and across the program’s opportunities to learn and between these and practicum experiences) was seen as high in the Philippines, Thailand, the Russian Federation, and the USA (among the countries with larger program samples); and in Botswana, Malaysia, Oman and Singapore.

Program Philosophy

What teachers think about mathematics as a subject and how it is best learned is an important area of concern across teacher education. Two scales representing these views had a clear link with the tests results, the view that mathematics is a collection of rules and procedures, and the view that mathematics is better learned through active learning (both scales are centered at 10 representing neutral). On average and consistent with widely accepted views on learner centered teaching (e.g., "teachers must focus on what the learner is thinking when learning—and not solely on the subject/lesson to be taught"), most secondary future teachers show a tendency to agree with the idea that mathematics requires inquiry oriented learning. There is less agreement (though still some support as most of the means are larger than 10) with the first view indicating that mathematics can be seen as a collection of rules and procedures, a view that if upheld would imply a more procedural view of mathematics and if rejected a more inquiry oriented view signaling a philosophy more attuned to current world thinking in education (see Tables 5 and 6).

Program Wealth

Programs' wealth is a scale created by measures of future teachers' socioeconomic status such as home possessions including number of books at home, and parents' levels of education. The wealthier programs are in the Russian Federation followed by the USA among the countries with larger samples (Table 5); and by Germany and Switzerland among the countries with smaller samples (see Table 6).

Future Teachers' Background

The findings on future secondary teachers' background reflect recruitment and selection policies in teacher education programs as well as the social and economic level of those who are attracted into teaching. Among future secondary teachers, those with higher socioeconomic status are in the Russian Federation and in the USA (in Table 5), and in Germany, Switzerland (in Table 6). Future secondary teachers are in their early to mid-twenties, with the younger group in the Philippines and the oldest in Germany. Most secondary future teachers are female, with the exception of Botswana, Chinese Taipei, Singapore and Switzerland (with proportions ranging from .38 to .48). Self-reported levels of attainment as per average grades in high school (with 1 "below average for year level," and 5 "always at top of year level") placed future secondary teachers as above average with those reporting higher grades in Oman, the Russian Federation and the USA (ranging from 4.65 to close to 3.5 out of 5) and those reporting the lowest grades in the Philippines (3.07). For the most part, teacher education policies vary regarding the quality of those that enter teacher education programs.

Future Teachers' Knowledge for Teaching Secondary Level Mathematics

To describe concretely individuals' levels of performance anchor points were developed. These were based on future secondary teachers' scores at specific points reached on the MCK and MPCK scales (Tatto et al., 2012, p. 135). Items used to describe performance at the anchor points were selected based on the probability that a future teacher with a score at that point would get the relevant items right. For MCK two anchor points were identified: *Anchor Point 1* representing a lower level of knowledge (with a 0.70 probability of answering the items correctly), and *Anchor Point 2*, representing a higher level of knowledge (with a 0.50 or less probability of answering the items correctly). For MPCK (a scale with two-third less items than MCK) one anchor point was identified. A panel of mathematicians and mathematics educators analyzed the items classified at these anchor points and formulated empirically based descriptions of the knowledge that future teachers demonstrated at each anchor point. The summary of the anchor point definitions is included in Table 2, and a detailed description of the anchor points and examples can be found in the study's international report (Tatto et al., 2012, p. 135-140). Anchor points are used in Tables 3 and 4 which show the MCK and MPCK results obtained by future secondary teachers across programs and countries as described next.

Mathematics content knowledge anchor points. For MCK, anchor point 1, representing a lower level of knowledge, corresponds to a scale score of 490. Anchor Point 2, representing a higher level of knowledge, corresponds to a scale score of 559. Table 3 shows for MCK, the proportion of teachers who scored above Anchor Point 1 and above Anchor Point 2, and the scaled score mean for the each of the programs groups found in the countries. For those future teachers prepared to teach the secondary curriculum up to grade 10 (designated as group 5 in Table 3), close to 87 percent of those in Singapore reach anchor point 1 but less than 40 percent reach anchor point 2. In Switzerland and Poland close to 75 and 80 percent reach anchor point 1 but again less than 40 percent reach anchor point 2. Only 53 percent in Germany and 33.5 percent in the U.S. reach anchor point 1, and only 12.6 and 2.1 percent of Germany and U.S. teachers respectively reach anchor point 2. The scores among those teaching secondary students in grade 11 and above (group 6 in Table 3) improve for countries that have both program types, and in the U.S. the proportion reaching anchor point 2 is significantly higher (almost 90 points). Specifically among the secondary programs preparing teachers to teach secondary up to grade 11 and above, over 90 percent of future secondary teachers in Chinese Taipei reach Anchor Point 2 (with a scale score of 667), and more than 60 percent in Singapore and Germany, but only 44.5 percent of those in the U.S. do so, nevertheless the average scale score between those U.S. future teachers in group 5 and group 6 is significantly different (468 versus 553 or 85 points in the assessment), and above the international mean.

Mathematics pedagogical content knowledge anchor point. One anchor point was defined for the secondary-level MPCK scale due to the comparatively smaller number of items measuring mathematics pedagogical content knowledge, representing a score of 509 on the scale (see Table 4). Overall these items were more challenging for all future secondary teachers, especially for those preparing to teach in the earlier grades of secondary school. Among those in programs preparing future secondary teachers to teach the curriculum up to grade 10 maximum (group 5), a larger proportion reached the anchor point in Switzerland, Singapore, Germany, and Poland (close to 71, 66, 52, and 50 percent respectively, with average scale scores ranging from 549 to 520) than the other countries. In the U.S. only 16.7 percent reach the anchor point (with an average scale score of 471). In group 6, corresponding to teachers prepared to teach the secondary curriculum to grade 11 and above, over 90 percent future secondary teachers reach the anchor point in Chinese Taipei (with an average scale of 649), followed by Germany, Singapore, and the Russian Federation (80, 75, 71 percent respectively). In the U.S. about 60 percent reach the anchor point (with an average scale score of 542), again showing the significantly different performances between teachers in group 5 and 6 (471 versus 542 or 71 points in the assessment), and above the international mean.

It should be noted that countries make quite different policy decisions on how to prepare future secondary teachers, and follow different implementation strategies. For instance in Chile, the Philippines and Switzerland there is only one program designed to prepare lower secondary teachers but with substantially different results across these three countries (with average scale scores of 354, 442, and 531 respectively); thus Switzerland by preparing "generalists" with "some specialization" de facto prepares teachers at a higher level than Chile also preparing "generalists" or than the Philippines preparing "specialists". In C. Taipei and Russia the policy is to prepare future secondary teachers to teach lower and upper secondary grades in one program and in this way manage to prepare highly knowledgeable teachers for both levels (with average scale scores of 667 and 594). In countries that have both programs it is easy to see the mediating role of program structure and design (including selection strategies) as all future teachers in group 6 do better on average than those in group 5.

Discussion

The focus on the TEDS-M results for secondary teacher education programs is important because mathematics at the secondary level can be seen as the gateway to mathematics based or related careers. The TEDS-M findings reveal a number of features that may be useful for future policy both systemic as well as programmatic. These features are: differentiation in teacher education offerings, the uneven outcomes of teacher education, and the variable quality and character of teacher education programs. I examine each one in turn.

Differentiation in teacher education offerings

The table 1 presented at the beginning of this paper shows the wide variability in the design of teacher education for secondary teachers. The most notable is the two strands that guide the secondary teacher education curriculum in some countries which seem to parallel the school curriculum. Lower secondary teachers are selected and prepared differently than those who are projected to teach higher secondary grades. While teacher education program's differentiation interacts with selection procedures, differential access to opportunities to learn key mathematics topics seems to be associated with lower scores in the TEDS-M assessments. Here it is important to note that it is access to opportunities to learn before and during the program that makes a difference rather than program structure yet these two dimensions in some cases interact as well.

Uneven outcomes of teacher education

An important finding from TEDS-M is that across countries secondary teacher certification may mean different things. The TEDS-M assessments were developed in collaboration with all the participating countries' teacher educators and education officials. The items were validated across countries and there was agreement that what the assessment measured were desirable knowledge outcomes for the teaching profession. The assessments were designed to align with the more ambitious goal of engaging mathematics as a process of inquiry as opposed to only mastering mathematical procedures. The variability found in the TEDS-M assessments reveals the degree to which future teachers were able to demonstrate the knowledge required to do so. The assessments results in some cases span one or two standard deviations between the countries and reveal an important policy challenge: namely that if the idea is to provide each child equitable access to mathematics education it is imperative that their teachers also receive equitable access. The data on SES and gender as well as former opportunities to learn reveal fundamental inequities in education systems that produce teacher candidates. Importantly however it is not only access to more mathematics but also the quality of the mathematics preparation received. TEDS-M shows that future teachers that espoused the view that mathematics is best learned by mastering a series of rules and procedures also had low scores in the TEDS-M assessments.

Variable quality of teacher education programs

The TEDS-M study also explored the degree to which teacher education programs were regulated and the association between higher scores in the assessments and the strength of their quality assurance systems / standards (Ingvarson et al., 2013). The finding was that the strength of the quality assurance mechanism established in each country (e.g., accreditation, program entry and exit standards, and monitoring performance during the program, successful induction) seemed to be associated with better outcomes.

While these findings are important the TEDS-M study is only the first study in a larger research program. The FIRSTMATH study is exploring whether those individuals who attain higher scores in our assessments are also the best teachers. FIRSTMATH just finished developing and testing instruments and methods with close to 15 countries. This preliminary proof-of-concept study has been successful and we are seeking funding for the main study which will also be international and comparative and will require representative samples of novice teachers.

For mathematics teacher educators it is imperative to engage in this kind of collaborative, research endeavor. Others have entered the field and are shaping policy in important and possibly negative ways. This seems to be a global phenomenon. For instance in the U.S. the secretary of education has called for high stakes regulations to teacher education released in November of last year (2014). After a period of commentary in January it is expected that the regulations will take effect this fall (2015). The regulations ask that states require their teacher education programs to collect and report information on direct and distant outcomes on the efficiency of their programs including work outcomes in the first 3 years after graduation, surveys of satisfaction from their graduates and employers, and proof of positive impact on the achievement of their pupils. There are request for accreditation according to standards (CAEP), and while programs have choices as to how they would like the accreditation procedure to occur accreditation is a must for survival. Programs will be ranked and low performing programs will be eliminated. While there is evidence from TEDS-M that quality assurance is associated with better program outcomes as reflected in our assessments, our knowledge is scarce concerning the dynamics of how these regulations may or may not improve programs. Further the requirement that teacher education programs prove impact on students' achievement is seen as a misguided policy as it makes assumptions that ignore the influence of school and pupil challenges that are not controlled by teacher education programs.

TEDS-M has developed the methods to assess outcomes and we are beginning the FIRSTMATH study to develop methods for a fair evaluation of the effectiveness of teacher education and of teachers. But these need to be non-high stakes, fair and useful to practitioners in order to improve practice. We have developed the methods, and done a field trial, you are invited to join us.

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Table 1

Organizational Characteristics of Teacher Education Program-Secondary types in the TEDS-M Study

Country	**Program-type**	**Consecutive/ Concurrent**	**Duration (years)**	**Grade span**	**Specialization**	**Program group**
Botswana	Diploma in Secondary Education, Colleges of Education	Concurrent	3	8-10	Specialist	**5:** Lower secondary (grade 10 max)
	Bachelor of Secondary Education (Science), University of Botswana	Concurrent	4	8-12	Specialist	**6:** Upper secondary (up to grade 11 and above)
Chile	Generalist	Concurrent	4	1-8	Generalist	**BOTH 3 (**Primary-lower secondary - grade 10 max) **& 5** (Lower secondary - grade 10 max)
	Generalist with further mathematics education	Concurrent	4	5-8	Generalist	**5:** Lower secondary (grade 10 max)
Chinese Taipei	Secondary Mathematics Teacher Education	Concurrent	4.5	7-12	Specialist	**6:** Upper secondary (up to grade 11 and above)
Germany	Teachers of Grades 1-9/10 with Mathematics as Teaching Subject (Type 2a)	Hybrid of the two	3.5+2.0	1-9/10	Specialist (in 2 subjects)	**BOTH 4 (**Primary mathematics specialist) **& 5** (Lower secondary - grade 10 max)
	Teachers for Grades 5/7-9/10 with Mathematics as Teaching Subject (Type 3)	Hybrid of the two	3.5 +2.0	5/7-9/10	Specialist (in 2 subjects)	**5:** Lower secondary (grade 10 max)
	Teachers for Grades 5/7-12/13 with Mathematics as a Teaching Subject (Type 4)	Hybrid of the two	4.5+2.0	5/7-12/13	Specialist (in 2 subjects)	**6:** Upper secondary (up to grade 11 and above)

Malaysia	B.Ed (Mathematics) Secondary	Concurrent	4	7-13	Specialist (in 2 subjects)	**6:** Upper secondary (up to grade 11 and above)
	B.Sc.Ed (Mathematics) Secondary	Concurrent	4	7-13	Specialist (in 2 subjects)	**6:** Upper secondary (up to grade 11 and above)
Oman	Bachelor of Education, University	Concurrent	5	5-12	Specialist	**6:** Upper secondary (up to grade 11 and above)
	Educational Diploma after B.Sc.	Consecutive	5+1	5-12	Specialist	**6:**Upper secondary (up to grade 11 and above)
	Bachelor of Education, Colleges of Education	Concurrent	4	5-12	Specialist	**6:**Upper secondary (up to grade 11 and above)
Philippines	Bachelor in Secondary Education	Concurrent	4	7-10	Specialist	**5:** Lower secondary (grade 10 max)
Poland	Mathematics BA First Cycle	Concurrent	3	4-9	Specialist	**BOTH 4 (**Primary mathematics specialist**) & 5** (Lower secondary - grade 10 max)
	Mathematics MA Long Cycle	Concurrent	5	4-12	Specialist	**BOTH 4 (**Primary math specialist) **& 6 (**Upper secondary - up to grade 11 and above)
Russian Federation	Teacher of Mathematics	Concurrent	5	5-11	Specialist	**6:** Upper secondary (up to grade 11 and above)
Singapore	PGDE, Lower Secondary	Consecutive	4+1	7-8	Specialist (in 2 subjects)	**5:** Lower secondary (grade 10 max)
	PGDE, Secondary	Consecutive	4+1	7-12	Specialist (in 2 subjects)	**6:** Upper secondary (up to grade 11 and above)
Switzerland	Teachers for Secondary School (Grades 7-9)	Concurrent	4.5	7-9	Generalist, some specialization	**5:** Lower secondary (grade 10 max)
Thailand	Bachelor of Education	Concurrent	5	1-12	Specialist	**BOTH 4 (**Primary mathematics specialist) **& 6** (Upper secondary - up to grade 11 and above)
	Graduate Diploma in Teaching Profession	Consecutive	4+1	1-12	Specialist	**BOTH 4 (**Primary math specialist) **& 6** (Upper secondary - up to grade 11 and above)

USA	Primary + Secondary Concurrent	Concurrent	4	4/5-8/9	Specialist	**BOTH 4 (**Primary mathematics specialist) **& 5** (Lower secondary - grade 10 max)
	Primary + Secondary Consecutive	Consecutive	4+1	4/5-8/9	Specialist	**BOTH 4 (**Primary mathematics specialist) **& 5** (Lower secondary - grade 10 max)
	Secondary Concurrent	Concurrent	4	6/7-12	Specialist	**6:** Upper secondary (up to grade 11 and above)
	Secondary Consecutive	Consecutive	4+1	6/7-12	Specialist	**6:** Upper secondary (up to grade 11 and above)

Source: Tatto et al., 2012.

Table 2
Summary of Anchor Point Descriptions for Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) for Secondary Future Teachers

	Mathematics Content Knowledge (MCK)		**Mathematics Pedagogical Content Knowledge (MPCK)**	
	Success with	***Difficulty with***	***Success with***	***Difficulty with***
AP 1 For MCK two anchor points were defined AP1 corre-sponds to a scale score of 490 For MPCK one anchor point was defined and corre-sponds to a scale score of 509	Demonstrating knowledge of con-cepts related to whole numbers, in-tegers, and rational numbers, and the as-sociated computa-tions	Describing general patterns	Knowing the lower-secondary curri-cu-lum, and plan-ning for instruction (for instance identi-fying prerequisites for teaching a deri-vation of the quad-ratic formula, and determining conse-quences of moving the concept of square root from the lower-secondary to the upper-secondary school mathematics curriculum)	Identifying or ana-lyzing errors in more complex mathematical situa-tions (for instance they could not con-sistently apply a ru-bric with descrip-tions of three per-formance levels to evaluate students' solutions to a prob-lem about linear and non-linear growth).
	Evaluating algebraic expressions cor-rectly, and solving simple linear and quadratic equations, particularly those that can be solved by substitution or trial and error.	Solving multi-step problems with com-plex linguistic or mathematical rela-tions	Enacting (teaching) school mathematics skills and evaluat-ing students' math-ematical work cor-rectly in some situa-tions (for instance determining if a stu-dent's diagram sat-isfied certain given conditions in geom-etry, and recogniz-ing a student's cor-rect argument about divisibility of whole numbers)	Understanding and interpreting stu-dents' thinking or determining appro-priate responses to students
	Demonstrating knowledge of stand-ard geometric fig-ures in the plane and space, and identify-ing and applying simple relations in plane geometry	Relating equivalent representations of concepts with a ten-dency to overgener-alize concepts	Analyzing students' errors when the stu-dents' work in-volved a single step or short explana-tions (for example, identifying an error in a histogram)	Understanding the concept and mean-ing of a valid math-ematical argument (for instance unable to evaluate invalid arguments such as recognizing that ex-amples are not suf-ficient to constitute a proof)

	Interpreting and solving more complex problems about numbers, algebra, and geometry if the context or problem type was commonly taught in lower-secondary schools.	Reasoning mathematically, particularly difficulty in recognizing faulty arguments and justifying or proving conclusions		
AP2 For MCK only corresponds to a scale score of 559	[likely to correctly do all the mathematics that could be done by a future teacher at Anchor Point 1]	Solving problems stated in purely abstract terms,		
	Knowing functions (particularly linear, quadratic, and exponential)	Working competently on foundational material, such as axiomatic systems		
	Reading, analyzing and applying abstract definitions and notation,	Reasoning logically (e.g., not attending to all conditions of definitions or theorems and confusing the truth of a statement with the validity of an argument)		
	Making and recognizing simple arguments	Recognizing valid proofs of more complex statements		
	Knowing some definitions and theorems typically taught in tertiary-level courses, such as calculus, abstract algebra, and college geometry, and applying them in straightforward situations.	Constructing and completing mathematical proofs		

Source: Tatto et al., 2012, p. 142-148.

Table 3

Future Secondary Teachers Mathematics Content Knowledge by Program Type and Country

Program Group	Country	Sample Size	Valid Data (N)	Percent Missing (Weighted)	Percent at or above Anchor Point 1 (SE)		Percent at or above Anchor Point 2 (SE)		Scaled Score: Mean (SE)	
Group 5. Lower Secondary (to Grade 10 Maximum)	**Botswana** [a]	34	34	0.0	6.0	(4.2)	0.0		436	(7)
	Chile [b]	746	741	0.6	1.2	(0.4)	0.0		354	(3)
	Germany	408	406	0.3	53.5	(3.4)	12.6	(2.2)	483	(5)
	Philippines	733	733	0.0	14.0	(3.0)	0.2	(0.1)	442	(5)
	Poland [c]	158	158	0.0	75.6	(3.5)	34.7	(3.2)	529	(4)
	Singapore	142	142	0.0	86.9	(3.1)	36.6	(4.3)	544	(4)
	Switzerland [d]	141	141	0.0	79.7	(3.4)	26.7	(3.2)	531	(4)
	† USA [f]	169	121	32.7	33.5	(2.2)	2.1	(1.3)	468	(4)
Group 6. Lower & Upper Secondary (to Grade 11 and above)	**Botswana** [a]	19	19	0.0	21.1	(7.4)	0.0		449	(8)
	Chinese Taipei	365	365	0.0	98.6	(0.8)	95.6	(1.0)	667	(4)
	Germany	363	362	0.1	93.4	(1.5)	62.1	(2.9)	585	(4)
	Malaysia	389	388	0.2	57.1	(2.3)	6.9	(0.9)	493	(2)
	Oman	268	268	0.0	37.1	(2.7)	1.8	(0.6)	472	(2)
	Poland	140	139	0.8	85.7	(2.6)	35.7	(2.7)	549	(4)
	Russian Federation [h]	2141	2139	0.1	88.8	(1.7)	61.1	(4.3)	594	(13)
	Singapore	251	251	0.0	97.6	(1.0)	62.9	(2.6)	587	(4)
	Thailand	652	652	0.0	41.0	(1.5)	8.4	(1.1)	479	(2)
	† USA [f]	438	354.0	21.3	87.1	(2.0)	44.5	(3.9)	553	(5)

Notes:

The dagger symbol (†) is used to alert readers to situations where data were available from less than 85% of respondents. The shaded areas identify data that, for reasons explained in the annotations, can be compared with data from other countries with caution. The solid vertical lines on the chart show the two Anchor Points (490 and 559).

a. Botswana: The sample size is small (N=53), but arises from a census of a small population. b. Chile: Combined participation rate between 60 and 75%. c. Poland: Reduced coverage: institutions with consecutive programs only were not covered. Combined participation rate between 60 and 75%. d. Switzerland: Reduced coverage: includes only institutions where German is the primary language of use and instruction. f. USA: Reduced coverage: public institutions only. Combined participation rate between 60% and 75%. An exception was made to accept data from one institution because one additional participant would have brought the response rate above the 50% threshold. Although the participation rate for the complete sample meets the required standards, the data contain records that were completed using a telephone interview, when circumstances did not allow administration of the full questionnaire. Of the 607 recorded as participants, the full questionnaire was administered to 502. Bias may arise in the data because significant numbers of individuals were not administered the full questionnaire. h. Russian Federation: An unknown number of those surveyed had previously qualified to become primary teachers.

Source: Tatto et al., 2012.

Table 4

Future Secondary Teachers Mathematics Pedagogical Content Knowledge by Program Type and Country

Program Group	Country	Sam-ple Size	Valid Data (N)	Percent Missing (Weighted)	Percent at or above Anchor Point (SE)		Scaled Score: Mean (SE)	
Group 5. Lower Sec-ondary (to Grade 10 Maximum)	**Botswana** [a]	34	34	0.0	8.9	(5.1)	436	(9)
	Chile [b]	746	741	0.6	5.7	(1.1)	394	(4)
	Germany	408	406	0.3	52.5	(4.6)	515	(6)
	Philippines	733	733	0.0	12.3	(2.0)	450	(5)
	Poland [c]	158	158	0.0	49.7	(3.1)	520	(5)
	Singapore	142	142	0.0	65.9	(4.2)	539	(6)
	Switzerland [d]	141	141	0.0	70.9	(3.8)	549	(6)
	† USA [f]	169	121	32.7	16.7	(3.1)	471	(4)
Group 6. Lower & Up-per Secondary (to Grade 11 and above)	**Botswana a**	19	19	0.0	5.3	(7.4)	409	(16)
	Chinese Taipei	365	365	0.0	93.3	(1.5)	649	(5)
	Germany	363	362	0.1	80.3	(2.7)	586	(7)
	Malaysia	389	388	0.2	27.9	(2.5)	472	(3)
	Oman	268	268	0.0	29.8	(2.9)	474	(4)
	Poland	140	139	0.8	62.2	(4.7)	528	(6)
	Russian Federation [h]	2141	2139	0.1	71.0	(3.1)	566	(10)
	Singapore	251	251	0.0	75.3	(3.1)	562	(6)
	Thailand	652	652	0.0	28.4	(1.9)	476	(2)
	† USA [f]	438	354.0	21.3	61.0	(3.0)	542	(6)

Notes:
The dagger symbol (†) is used to alert readers to situations where data were available from less than 85% of respondents. The shaded areas identify data that, for reasons explained in the annotations, can be compared with data from other countries with caution. The solid vertical line on the chart shows the single Anchor Point (509). a. Botswana: The sample size is small (N=53), but arises from a census of a small population. b. Chile: Combined participation rate between 60 and 75%. c. Poland: Reduced coverage: institutions with consecutive programs only were not covered. Combined participation rate between 60 and 75%. d. Switzerland: Reduced coverage: includes only institutions where German is the primary language of use and instruction. f. USA: Reduced coverage: public institutions only. Combined participation rate between 60% and 75%. An exception was made to accept data from one institution because one additional participant would have brought the response rate above the 50% threshold. Although the participation rate for the complete sample meets the required standards, the data contain records that were completed using a telephone interview, when circumstances did not allow administration of the full questionnaire. Of the 607 recorded as participants, the full questionnaire was administered to 502. Bias may arise in the data because significant numbers of individuals were not administered the full questionnaire. h. Russian Federation: An unknown number of those surveyed had previously qualified to become primary teachers.

Source: Tatto et al., 2012.

Table 5

Means and Standard Deviations for TEDS-M Future Teachers Knowledge for Teaching Mathematics in Secondary Programs for Countries with Larger Program Samples

	Chile $N_{Level\ 1}$ =648 $N_{Level\ 2}$ =37		**Philippines** $N_{Level\ 1}$ =668 $N_{Level\ 2}$ =46		**Poland** $N_{Level\ 1}$ =247 $N_{Level\ 2}$ =34		**Russian Fed.** $N_{Level\ 1}$ =1951 $N_{Level\ 2}$ =48		**Thailand** $N_{Level\ 1}$ =614 $N_{Level\ 2}$ =52		**United States** $N_{Level\ 1}$ =461 $N_{Level\ 2}$ =68	
Future Teachers Secondary (Level 1)	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
MCK score	**356.35**	**84.60**	**449.36**	**48.29**	**536.05**	**88.91**	**593.26**	**90.84**	**478.46**	**57.91**	**536.13**	**65.36**
MPCK Score	**394.58**	**87.70**	**451.50**	**60.75**	**525.98**	**95.04**	**569.01**	**94.67**	**477.41**	**64.37**	**529.15**	**80.55**
SES*[SES]	**-0.29**	**0.80**	**-0.63**	**0.86**	**-0.11**	**0.73**	**0.60**	**0.64**	**-0.90**	**1.06**	**0.46**	**0.84**
Age [MFA001]	**23.85**	**2.80**	**20.96**	**2.00**	**23.13**	**5.33**	**22.01**	**1.59**	**22.34**	**0.81**	**25.26**	**6.45**
Proportion female [MFA002_]1=F; 0=M	**0.84**	**0.36**	**0.65**	**0.48**	**0.81**	**0.22**	**0.72**	**0.45**	**0.75**	**0.43**	**0.69**	**0.28**
Prior attainment: Average grades in secondary school (1=below average for year level; 5=Always at top of year level) [MFA009_]	**3.28**	**1.14**	**3.07**	**0.95**	**3.28**	**0.84**	**3.80**	**0.89**	**3.29**	**0.84**	**3.88**	**1.00**
Teacher Education Programs Secondary (Level 2)	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
Average number of university level mathematics topics in geometry ever studied (range 0-4) [MFB1GEOM]	**1.87**	**0.54**	**2.78**	**0.45**	**3.23**	**0.47**	**3.81**	**0.21**	**3.41**	**0.42**	**2.59**	**0.74**
Average number of school level mathematics topics in function, probability and calculus studied as part of the TE program (range 0-4) [MFB2SLMF]	**1.53**	**0.45**	**2.74**	**0.50**	**3.82**	**0.25**	**3.46**	**0.32**	**3.51**	**0.61**	**2.81**	**0.79**
Average frequency with which future teachers engaged in reading research on teaching and mathematics (scales centered at 10 representing neutral) [MFB5READ]	**9.22**	**0.96**	**10.68**	**0.74**	**8.15**	**1.34**	**10.28**	**0.75**	**10.31**	**0.75**	**10.61**	**1.34**
Average level of program coherence (scales centered at 10 representing neutral)[MFB15COH]	**11.90**	**1.21**	**13.59**	**0.88**	**11.53**	**1.14**	**12.93**	**0.75**	**13.01**	**0.99**	**12.78**	**1.63**

Average agreement with the belief that mathematics is a collection of rules and procedures (scales centered at 10 representing neutral) [MFD1RULE]	**10.94**	**0.55**	**12.67**	**0.63**	**10.39**	**0.51**	**10.52**	**0.28**	**11.83**	**0.56**	**10.71**	**0.59**
Average agreement with the belief that mathematics is better learned through active learning (scales centered at 10 representing neutral) [MFD2ACTV]	**12.73**	**0.48**	**11.80**	**0.50**	**12.29**	**0.79**	**11.89**	**0.47**	**11.96**	**0.55**	**12.11**	**0.90**
Average SES for each program (aggregated from future teachers SES) [SES]	**-0.21**	**0.47**	**-0.64**	**0.49**	**-0.10**	**0.28**	**0.60**	**0.17**	**-0.83**	**0.54**	**0.47**	**0.48**

Table 6

Means and Standard Deviations for TEDS-M Future Teachers in Secondary Programs for Countries with Smaller Program Samples

	BOTSWANA N=31		CHINESE TAIPEI N=355		GERMANY N=620		MALAYSIA N=357		OMAN N=153		SINGAPORE N=371		SWITZER-LAND N=137	
Future Teachers (Level 1)	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
MCK score	**437.48**	**41.88**	**666.58**	**75.37**	**541.91**	**84.33**	**495.06**	**51.09**	**470.96**	**46.84**	**573.92**	**60.72**	**530.64**	**48.79**
MPCK Score	**431.22**	**62.85**	**647.67**	**94.46**	**553.12**	**98.16**	**472.95**	**62.55**	**466.00**	**68.97**	**554.84**	**84.69**	**546.48**	**73.03**
SES*[Zscore: REGR factor score…]	**-1.30**	**.67**	**-.50**	**.88**	**.41**	**.94**	**-.69**	**.79**	**-1.05**	**.59**	**-.55**	**.79**	**.12**	**.91**
Age [MFA001]	**24.74**	**3.57**	**24.06**	**2.28**	**28.98**	**4.91**	**22.70**	**2.32**	**21.93**	**.856**	**26.73**	**4.00**	**26.20**	**4.30**
Proportion female [MFA002_]1=F; 0=M	**.39**	**.49**	**.38**	**.49**	**.62**	**.48**	**.82**	**.38**	**.60**	**.49**	**.48**	**.50**	**.42**	**.49**
Prior attainment: Average grades in secondary school (1=below average for year level; 5=Always at top of year level) [MFA009_]	**3.77**	**.76**	**3.66**	**1.07**	**3.32**	**.88**	**3.82**	**.96**	**4.65**	**.58**	**3.52**	**.95**	**3.41**	**.91**
Teacher Education Programs (Level 2)	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**	**M**	**SD**
Average number of university level mathematics topics in geometry ever studied (range 0-4) [MFB1GEOM]	**1.81**	**.30**	**3.23**	**.33**	**2.20**	**.47**	**2.77**	**.45**	**3.41**	**.26**	**1.49**	**.40**	**2.78**	**.43**
Average number of school level mathematics topics in function, probability and calculus studied as part of the TE program (range 0-4) [MFB2SLMF]	**3.00**	**.10**	**3.45**	**.35**	**2.48**	**.57**	**3.45**	**.17**	**3.30**	**.13**	**2.63**	**.30**	**2.90**	**.40**
Average frequency with which future teachers engaged in reading research on teaching and mathematics (scales centered at 10 representing neutral) [MFB5READ]	**10.55**	**.44**	**9.69**	**.86**	**8.01**	**.49**	**10.37**	**.30**	**10.04**	**.23**	**9.12**	**.15**	**8.75**	**.80**
Average level of program coherence (scales centered at 10 representing neutral)[MFB15COH]	**12.84**	**1.34**	**11.97**	**.57**	**9.17**	**.48**	**12.73**	**.50**	**12.45**	**1.04**	**12.03**	**.17**	**10/45**	**.87**
Average agreement with the belief that mathematics is a collection of rules and	**11.49**	**.17**	**10.81**	**.20**	**9.66**	**.15**	**11.63**	**.19**	**11.38**	**.25**	**10.91**	**.07**	**9.85**	**.28**

procedures (scales centered at 10 representing neutral) [MFD1RULE]														
Average agreement with the belief that mathematics is better learned through active learning (scales centered at 10 representing neutral) [MFD2ACTV]	**11.79**	**.21**	**12.35**	**.26**	**12.43**	**.33**	**11.38**	**.21**	**11.99**	**.34**	**11.53**	**.15**	**12.47**	**.43**
Average SES for each program (aggregated from future teachers SES) [SES]	**-1.37**	**.06**	**-.50**	**.20**	**.41**	**.30**	**-.70**	**.15**	**-1.05**	**.13**	**-.55**	**.11**	**.12**	**.20**