Professional Knowledge of (Prospective) Mathematics Teachers – Its Structure and Development

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Summary

Recent research on the professional knowledge of mathematics teachers, which has been carried out in the last decade, is in the focus of this paper. Building on the international IEA Teacher Education and Development Study – Learning to Teach Mathematics (TEDS-M), this paper describes a more situated way of evaluating the professional knowledge of teachers. The theoretical framework of the follow-up study of TEDS-M takes up the novice-expert framework and analyses via video-based assessment instruments the structure and development of the professional knowledge of mathematics teachers. More recent concepts on noticing and interpreting classroom situations and students' activities are also incorporated into the analysis. Connecting the results of the study TEDS-FU with the study TEDS-M gives insight into the development of the professional knowledge of mathematics teachers.

Keywords: empirical studies, international comparative studies, teacher education.

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Introduction

Studies on the knowledge of mathematics teachers have gained significant relevance in the last decade (for an overview see for example Blömeke & Delaney, 2012). Criticism about the inefficiency of teacher education has long been voiced in many Western countries. Teacher education in general has been described as a weak intervention compared to one's own school experience and later professional socialisation (Richardson, 1996). More particularly referring to mathematics teacher education, Klein (1932, German original 1908) criticised already at the beginning of the last century in his famous metaphor of a "double discontinuity" the lack of impact of university education on teaching practice in school.

In the light of the growing importance of international comparative studies on students' achievements in mathematics like TIMSS or PISA the professional knowledge of mathematics teachers and its influence on the development of the knowledge of students at school has become of special interest. The effectiveness of mathematics teacher education, i.e. the question how far universities succeed in the development of the professional knowledge of future mathematics teachers during their study, is a core question within this debate.

In the last decades a substantial number of national and international studies on mathematics teacher education have been carried out. As Krainer and Llinares (2010) pointed out in their comprehensive survey on the state-of-the-art on mathematics teacher education (MTE), three trends can be identified in the literature on mathematical learning of the three groups of prospective teachers, teachers, and teacher educators, namely "(1) teacher educators' and researchers' increasing attention to the social dimension and (2) attention to teachers' reflections" (p. 702).

The first trend including the social dimension of mathematics teacher education incorporates a shift from the perspective of the training of individual future teachers and teachers to practice and research emphasising the social dimension in teacher education has led to a strong change in the discussion on teacher education. For example, Krainer and Llinares (2010) point out that the concepts of collaborative learning, teacher-inquiry groups, communities of practice have played an important role in the recent discussion on mathematics teacher education, which is reflected in a strong shift towards the inclusion of sociological and sociocultural theories in research papers in the conference proceedings of PME.

The second trend, with a focus on teachers' reflective practice, is partially connected with the social shift described above and refers to the growth of teachers as professionals. For example, the research developed in the last decade on teachers' noticing when they observe their classes, how they interpret the observations made and how these interpretations change their practice, belongs to this developing aspect of research. The third trend described by Krainer and Llinares as

> "increasing attention to the general conditions of teacher education (e.g., time, structure, institutional settings, and human resources), is newer and can be seen as an influence of work done on the practice and research in MTE in other fields, for example, organizational development" (p. 702).

Krainer and Llinares (2010) make a strong plea for

> "taking these three trends seriously and regarding them as the challenges for the future"

> (p. 704). They comment that a further challenge is the fact that many studies on mathematics teacher education use qualitative research methods and argue that "more external and quantitative research are needed, in particular, looking at the outcomes of different types of teacher education or at longitudinal studies of mathematics teachers' learning and career. In all these cases, large populations are necessary to test relevant hypotheses" (p. 705).

They describe the creation of competence models for prospective teachers as challenge for the future in order to analyse different kinds of knowledge of teachers and prospective teachers. Referring to the work by Adler et al. (2005) they state: "Overall, there is a future challenge to combine qualitative and quantitative research methods and to integrate systematic reflections of teachers into research projects" (p. 705).

This research-oriented view on mathematics teacher education and student achievement is complemented by discussions in the light of international comparative studies. Such studies yield constantly strong differences in mathematics achievement between East Asian and Western students. Based on the results of large-scale studies like TIMSS or PISA, Leung and Park (2002) ask the question, whether the "competence of the East Asian students can be attributed at least partly to the competence of their teachers" (p. 128). This assertion leads to the question whether in teacher education the same achievement differences between Eastern and Western students prevailing over the last two decades are valid for prospective teachers as well and if yes, how far different systems of teacher education lead to these achievement differences.

The questions of how effective different educational systems on mathematics teachers are, and to what extent do country-specific differences exist, has lead the International Association for the Evaluation of Educational Achievement (IEA) to implement an international study on the effectiveness of teacher education at primary and lower secondary level, the so-called "Teacher Education and Development Study – Learning to Teach Mathematics (TEDS-M)" (see Tatto et al., 2008) in the last decade. In the following sections, an overview on the discussion of the professional knowledge of (future) mathematics teachers will be presented including the TEDS-M study on teacher education and a follow-up study on the professional knowledge of practising teachers, the so-called TEDS-FU study in which the transition of mathematics teachers from teacher education into the profession is examined.

Survey on the professional knowledge of (prospective) mathematics teachers

In their comprehensive survey on the state of research on the assessment of teacher knowledge across countries, Blömeke and Delaney (2012) point out that warning signs exist about the low proficiency levels of mathematics teachers in Western countries. However, prior to TEDS-M there appeared to be no systematic evidence on the state of these proficiencies. Since the late 1990s several small-scale comparative studies on mathematics teacher education and its efficiency have been carried out (cf. Ma, 1999). The survey, presented at ICME-10 in Copenhagen (Adler et al., 2005), the 15th ICMI Study (Even & Ball, 2008) and published in the International Handbook of Mathematics Teacher Education (Wood, 2008), provided a huge step forward and had the potential to fill many gaps in research concerning the efficiency of mathematics teacher education. Concerning the knowledge domain, the scope of these studies was limited, as many of these studies were either case studies or based on self-reports. Other studies did not include the knowledge domain and focused instead on beliefs or other concepts. To summarise the state of research prior to TEDS-M, we refer in the following to the extensive survey by Blömeke and Delaney (2012) on the professional knowledge of (prospective) mathematics teachers and restrict ourselves to a few selected results (for details see Blömeke & Delaney, 2012).

In the area of the professional knowledge of prospective mathematics teachers earlier work characterised pre-service teacher education as teacher learning, understanding teacher education as a kind of an apprenticeship. The 1990s have then seen a growing number of empirical studies on mathematics teacher education. However, many of these studies were conducted within their own education institution (cf. Chick et al.,), which implied several limitations as Adler et al. (2005) point out. Further research on teacher education turned more strongly to the knowledge base of teachers' classroom practice and developed theoretical conceptualisations in close relation to teaching practice (cf. the studies contained in the book edited by Rowland & Ruthven, 2010).

More recent studies are on the one hand similar to the studies described above, but are on the other hand characterised by a more analytical approach of defining and distinguishing between different knowledge facets functional for teaching and stressing the importance of mathematics content knowledge. These studies depart from a notion of competency related to competency-oriented approaches in international comparative studies on students' achievements such as PISA. Modelling the resources for proficiency in teaching mathematics in a multi-dimensional way is one important source for the theoretical framework as it has been described by Schoenfeld and Kilpatrick (2008) and further developed by Schoenfeld (2011), who sees teaching as a knowledge-intensive domain with different knowledge and affective-motivational facets.

Several large-scale studies on mathematics teacher knowledge share this common theoretical orientation, the already mentioned TEDS-M study, which will be described in detail in the next chapter, the study Mathematical Knowledge for Teaching (MKT), developed by the Learning Mathematics for Teaching Group of researchers from the University of Michigan (Ball & Bass, 2000) and the Cognitive Activation in the Classroom Project (COACTIV) developed by German researchers (Kunter et al., 2013). While TEDS-M and COACTIV are linked to the seminal classification of the different facets of professional knowledge of teachers developed by Shulman (1986), the MKT framework was inspired by Shulman's idea of pedagogical content knowledge and categorises the domains of knowledge needed to teach (see Ball et al., 2008). The COACTIV as well as the MKT study connect the professional knowledge of teachers with the growth of students' mathematical achievements, which is not the case with TEDS-M. The focus of TEDS-M is on an international comparison of the professional knowledge of prospective teachers for primary and secondary level, thus examining how their knowledge can be fostered during teacher education in contrast to the other two studies.

Apart from these differences it can be summarised that research on the professional knowledge of prospective teachers has increased dramatically with many small-scale and a few large-scale studies. These studies develop different descriptions of the structure of the professional knowledge of prospective teachers as they distinguish different facets of the knowledge base, including affective aspects such as the belief systems of the teachers. The common core of most studies can be aligned with the description of pedagogical content knowledge (PCK) of teachers following Shulman's (1987) seminal work in which PCK is defined as "that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special from of professional understanding" (p. 8). In their extensive survey on the current discussion around PCK, Depaepe et al. (2013) point out the special importance of this concept used by many studies.

However, despite the general agreement on PCK as connection between content and pedagogy and its dependence on the particular subject matter, no general consensus exists in empirical research on the facets of this important concept. Further, Depaepe et al. (2013) argue that there is an important group of empirical studies that do not define any component of PCK, although PCK was the central topic of this group of studies. Their study revealed consequences of the ongoing debate on the two principally different views on the conceptualisation of PCK, namely “whether mathematical knowledge in teaching is located ‘in the head’ of the individual teacher or is somehow a social asset, meaningful only in the context of its applications” (Rowland & Ruthven, 2011, p. 3).

Adherents of the cognitive perspective define according to Depaepe et al. (2013)

> “– in line with Shulman – a limited number of components to be part of PCK and distinguish PCK from other categories of teachers’ knowledge base, such as content knowledge and general pedagogical knowledge. By contrast, proponents of a situated perspective on PCK as knowing-to-act within a particular classroom context, typically acknowledge that the act of teaching is multi-dimensional in nature and that teachers’ choices simultaneously reflect mathematical and pedagogical deliberations” (p. 22).

These paradigmatic differences in the conceptualisations of PCK have, according to Depaepe et al. (2013), an impact on the way in which PCK is empirically investigated.

> “Advocates of a cognitive perspective on PCK believe it can be measured independently from the classroom context in which it is used, most often through a test. They typically focus on gaps in individual teachers’ PCK, on how PCK is related to and distinguished from other categories of teachers’ knowledge base …. Adherents of a situated perspective on PCK, on the contrary, typically assume that investigating PCK only makes sense within the context in which it is enacted. Therefore, they often rely on classroom observations (in some cases supplemented with other data sources such as interviews, lesson plans, logbooks) …” (p. 22)

The analyses by Depaepe et al. (2013) characterise the paradigmatic disagreement among scholars on the way how to conceptualise and evaluate teachers’ professional knowledge, including PCK, within different perspectives. Depaepe et al. (2013) conclude by calling for the integration of the cognitive perspective and the situated perspective, because both perspectives have their pitfalls, for example, neglecting the socio-cultural background of teaching or ignoring of the interactions of different knowledge categories within the cognitive perspective. Both perspectives provide powerful insights into teacher professional knowledge and so should be harnessed in a way that furthers understanding of how this aspect of teacher education influences teaching and learning.

In the following we will describe the results of the TEDS-M study and its continuation in TEDS-FU in order to show, how both kinds of research can be integrated.

Design and structure of TEDS-M

The comparative “Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)”, carried out under the auspices of the International Association for the Evaluation of Educational Achievement (IEA), evaluated the effectiveness of teacher education in terms of teacher knowledge and teacher beliefs both across countries and subject-specifically for the first time (for an overview see Blömeke et al., 2014; Tatto et al., 2008). TEDS-M was the first large-scale assessment of higher education that included direct testing covering graduates from 16 countries from East and West. The study includes a primary study and a lower-secondary study. The focus of TEDS-M were prospective teachers in their final year of teacher education who would receive a licence to teach mathematics in one of the grade 1 through 4 (primary study) or in grade 8 (lower-secondary study). The two studies were based on nationally representative samples and had to follow the rigorous IEA quality control mechanisms of sampling, data collection, coding, and data analysis. About 23,000 prospective teachers participated in the two studies, which took place from 2007-2009, the results were released in 2010.

The main questions of TEDS-M were multi-layered, namely as follows:

1. What are the professional competencies of future mathematics teachers?
2. How distinctive are the institutional conditions of mathematics teacher education?
3. What are the national conditions of mathematics teacher education?

We will limit ourselves in the following on the first question. Because teaching is the core task of teachers, and thus the development of teaching abilities internationally constitutes the main function of teacher education, teaching abilities – called 'professional competencies'– are the starting point of the theoretical framework of TEDS-M. According to Weinert (2001), professional competencies can be divided up into cognitive facets (in our context, teachers' professional knowledge) and affective-motivational facets (in our context, e.g., professional beliefs). The professional knowledge of teachers can again be divided into several facets. Referring to Shulman (1986), the following facets were distinguished in TEDS-M: mathematics content knowledge (MCK), mathematics pedagogical content knowledge (MPCK), including curricular knowledge, and general pedagogical knowledge (GPK).

TEDS-M examined also the professional beliefs held by the future teachers, due to the fact that beliefs are crucial for the perception of classroom situations and for decisions how to act, as Schoenfeld (2011) pointed out. Based on Richardson (1996), beliefs can be defined as stable, psychologically held propositions of the world around us, which are accepted to be true. In TEDS-M, several belief facets were distinguished, in particular epistemological beliefs about the nature of mathematics and beliefs about the teaching and learning of mathematics (Thompson, 1992). In addition, beliefs and affective traits such as motivation, and also metacognitive abilities

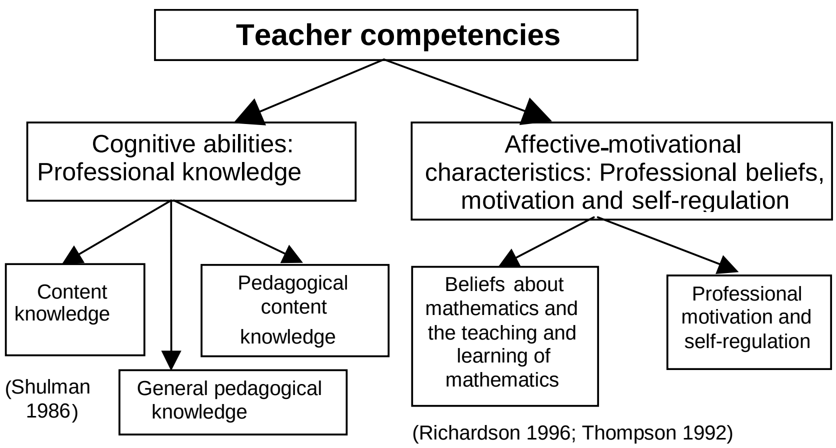

Figure 1. Conceptual model of teachers' professional competencies

such as self-regulation, are indispensable parts of the professional competencies of teachers (as displayed in fig. 1).

These facets of professional knowledge are further differentiated: mathematical content knowledge covers the main mathematical areas relevant for future teachers, mathematics pedagogical content knowledge covers curricular knowledge, knowledge of lesson planning and interactive knowledge applied to teaching situations (see fig. 2).

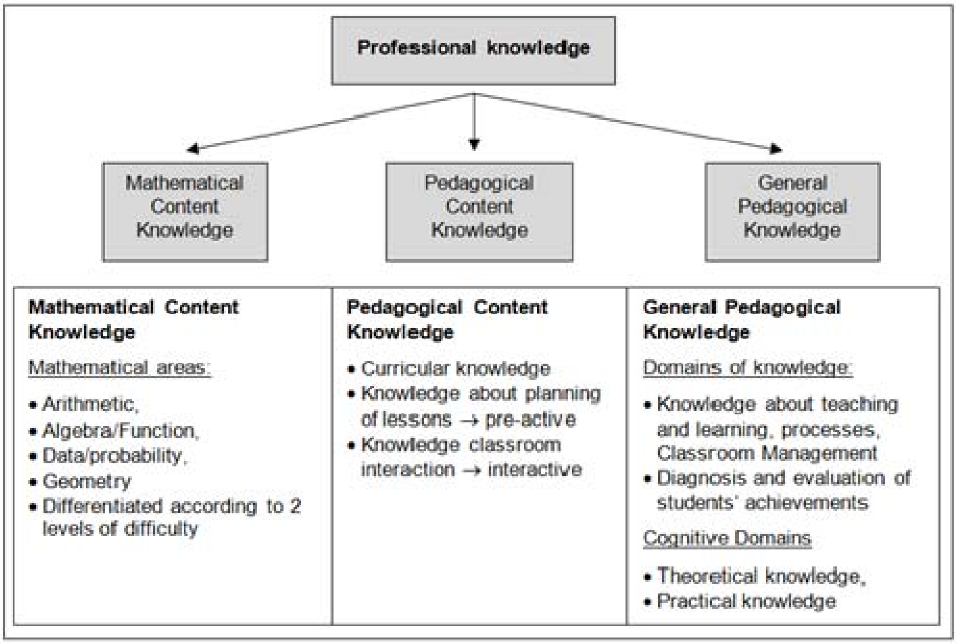

Figure 2. TEDS-M model of professional knowledge (Tatto et al., 2008)

TEDS-M examined the effectiveness of mathematics teacher education using the instruments of a future teacher survey, teacher educator survey, expert survey, document analysis of a sample of course offerings. The cognitive and affective-motivational facets of the future teachers' competencies were measured as criteria for effective teacher education. The future teachers' MCK and PCK were assessed in every participating country of TEDS-M, as well as their subject-related beliefs and professional motivations. Germany, Chinese Taipei and the USA assessed the GPK in a supplementary study using an instrument developed by König et al. (2011). Metacognitive abilities, however, were not part of the TEDS-M surveys.

Due to space limitations we cannot describe item examples, but refer to the extensive descriptions in Blömeke et al. (2014) and ZDM – The International Journal on Mathematics education, issue 3 in 2012.

Professional knowledge of prospective mathematics teachers – results of TEDS-M

The results of TEDS-M on the prospective teachers' achievement revealed huge differences between the participating countries, both concerning MCK and MPCK. In the primary study the participants from Chinese Taipei and Singapore showed the highest performance in MCK, significantly distinct to the performance of the other participating countries. The results of prospective teachers from USA and Germany were marginally above the international mean, the difference to the achievement of future teachers from Chinese Taipei and Singapore added to approximately one standard deviation. The achievement of future teachers from USA and Germany was not only lower than those of the future East Asian teachers, they were also significantly lower than the future teachers from Switzerland. Concerning MPCK, the performance pattern was quite similar: The future primary teachers from Singapore and Chinese Taipei achieved much higher test results than the future teachers from the other countries. German students' attainments were around the international mean, the difference from the students' achievements of Singapore and Chinese Taipei was again about one standard deviation. In addition, the MPCK results from the German students were significantly lower than the attainments from the students from Switzerland, the USA and Norway.

In the secondary study, participants from Chinese Taipei outperformed all other participants, in relation to MCK as well as MPCK. Participants from Russia, Singapore, Poland and Switzerland followed the Chinese Taipei prospective teachers with their achievements in MCK, German and US American prospective teachers achieved slightly above the average, whereas in relation to MPCK, prospective teachers from Russia, Singapore, Switzerland, Germany and Poland achieved the highest results after the Chinese Taipei participants, with prospective teachers from the USA close to the international mean. These results point to interesting differences between prospective teachers for primary level and secondary level and confirm the superior performance of Eastern prospective teachers compared to their Western counterparts in most areas. This is consistent with the achievement differences at student level in respective countries (for details see amongst others the comprehensive overview on the TEDS-M results in Blömeke et al., 2014 and Tatto et al., 2012).

A comparison of the relative strengths and weaknesses in MPCK and MCK (using ipsative values) reveal interesting results. Comparing the achievements of the prospective primary teachers country-wise in the area of MCK and MPCK allow to develop country specific achievement profiles:

- Relatively strong achievement in MCK compared to international mean differences between MCK and MPCK – from Asia, the prospective teachers from Chinese Taipei and Thailand belong to this group, from East and Middle Europe the future teachers from Russia, Poland, Germany and Switzerland can be assigned to this group.
- Relatively strong achievement in MPCK compared to international mean differences between MCK and MPCK – several Eastern and Western countries contribute to this cohort, namely the future teachers from Norway, the USA, Spain, Chile, Malaysia, and the Philippines.
- Knowledge relatively levelled and close to international mean differences between MCK and MPCK – one East Asian country, namely Singapore, and one country from the former Soviet Union, namely Georgia, belong to this group as well as Botswana.

The absolute level of achievement does not influence this pattern, apparently neither a particularly strong emphasis on MCK nor on MPCK supports the overall achievement of the prospective teachers of a country. It is remarkable that the two East Asian countries belong to different groups, although cultural traditions seem to have influenced this diverse pattern. The tradition of Confucianism in East Asian countries, labelled as Confucian Heritage Culture (CHC), sees the teacher as an expert, who possesses the content knowledge students need to acquire. This tradition leads to a high importance of content knowledge in teacher education in many East Asian countries. In Continental Europe, content-related approaches also place traditionally high emphasis on knowledge strongly connected to content-related reflections but this within PCK (being one strand within the European didactics traditions), which explains the high importance of content knowledge in Germany and Switzerland. Eastern European countries have historical roots linked to the Continental European educational systems including teacher education, content knowledge and content-related didactics, which is reflected in the high importance of MCK in Russia and Poland. These very different traditions may have led to the relatively high level of MCK compared to MPCK of the future teachers from East Asian and East European countries.

In contrast, in Scandinavian countries, North and South America, and in countries shaped by US-American influence such as the Philippines or Singapore a so called "progressive education" with child-centred approaches characterises school and teacher education are employed. These traditions may have led to the high level of MPCK compared to MCK of the future teachers from Scandinavian and American countries (for details see Kaiser & Blömeke, 2013). The situation is even more varied for prospective teachers for secondary level, which shows the strong, but not exclusive dominating influence of culture on education.

In further analyses going beyond country means, country-specific strengths and weaknesses in the knowledge of prospective teachers were detected by using differential item functioning (DIF). The item-by-item analyses reveals that due to differences in the cultural context, teachers from different countries responded differently to subgroups of test items with certain characteristics such as those stemming from certain particular domains, requiring similar cognitive demands or using the same item format. The analyses show that prospective teachers from Chinese Taipei and Singapore were particularly strong on mathematics content and constructed-response items. Prospective teachers from Russia and Poland were particularly strong on items requiring nonstandard mathematical operations. The USA and Norway achieved strongly on mathematics pedagogical content and data items. These results point once more to the influences of the cultural context on mathematics teacher knowledge.

Cultural influences on the results of TEDS-M cannot only be seen at the achievement level, but also in the area of the future teachers' beliefs. TEDS-M has evaluated in detail epistemological beliefs on the nature of mathematics and on the genesis of mathematical knowledge, i.e. the nature of mathematics teaching and learning. The studies explore amongst others the extent to which a country's culture can be characterised by an individualistic versus a collectivistic orientation using the cultural-sociological theory of Hofstede (1986). The collectivism-individualism antagonism describes the extent to which the individuals of a society are perceived as autonomous, the role and the responsibility of the individual for knowledge acquisition plays an important role.

The analyses (based on ipsative values) show that prospective teachers from more collectivistic-oriented countries such as Malaysia, Russia, Thailand, and the Philippines agree much more strongly to static aspects of mathematics (seeing mathematics as theory and a set of rules) in relation to dynamic aspects (describing mathematics as process to develop new mathematics insight) than it happened on average across the participating countries. In contrast, prospective teachers from highly individualistic countries such as Norway, Switzerland, and Germany much more strongly emphasised the dynamic nature of mathematics. Prospective teachers from countries that cannot be characterised as individualistic or collectivistic, namely Spain, Chinese Taipei, and Singapore, emphasised both aspects of mathematics in line with the international average (for details see Blömeke et al., 2014).

Currently, the question of the effectiveness of mathematics teacher education is of great interest. Disappointing first results demonstrate the limited influence of MPCK courses on the development of teacher professional knowledge (Blömeke et al., 2011) although this could be mitigated by a more differentiated and more extensive analyses. Internationally it was possible to identify two teacher profiles at the end of pre-service courses: teachers with a cognitively demanding and dynamic-constructivist accented competence profile and teachers with a lower achieving competence profile with more static and transmission-oriented beliefs. As explanatory features of the assignment to the profile the aspects gender, MCK and MPCK opportunities to learn as well as the coherence of the education could be identified.

The results lead to direct consequences for possible reform processes in teacher education. Furthermore, the high explanatory power of opportunities to learn in MPCK is of high relevance. These results lead for the first time to different conclusions regarding the importance of the different opportunities to learn: former analyses emphasized mathematics as predictive instance for the different educational attainment results. Looking at teacher competence as a multidimensional construct, the influential effect of MPCK courses come into the foreground (Blömeke et al., 2012). More important results of TEDS-M can be found in relevant journals or in Blömeke et al. (2014).

Design and structure of TEDS-FU

In the follow-up study of TEDS-M, TEDS-FU, the question of how mathematics teachers' professional knowledge develops after the end of teacher education in the first years of their school career based on the framework and the instruments of TEDS-M is explored. In addition, it is examined how professional knowledge can be analysed in a more performance-oriented way and how teacher expertise develops. Building on work from expertise research (for a review, see Li & Kaiser, 2011), professional competence of teachers is characterized by a high degree of integration of knowledge with multiple links, a modified categorical perception of teaching situations and by increasing integration of the different dimensions of professional knowledge. From the perspective of MPCK, this means an increase in conceptual understanding, the differentiation of a repertoire of heuristic strategies and metacognitive control strategies, an increasing competence through teaching and an increase in knowledge of school mathematics in depth and width (Schoenfeld & Kilpatrick, 2008).

In addition to MCK, MPCK and GPK as central cognitive facets of the professional competence of teachers the following practice-oriented, situated indicators of teacher expertise were considered: the precise perception of different mathematical classroom situations, described as perception accuracy or "noticing" (Van Es & Sherin, 2002) under the perspective of "selective attention" (Sherin, 2007) and their adequate analysis and interpretation as well as the flexible reaction on it, described as "knowledge-based reasoning" (Sherin, 2007). Due to the high importance of speed within the teaching profession we identify as further indicator for teacher expertise the fast recognition of mathematical student errors. Research on expertise points out that fast and adequate identification of errors is indeed a measure for differences in expertise level.

In the study TEDS-FU, carried out from 2010 to 2013, participants from the TEDS-M primary and secondary study were tested on a voluntary basis. The tests were web-based and the professional knowledge of the teachers was evaluated using video vignettes with short teaching sequences dealing amongst others with effective classroom management, heterogeneity, individualisation, teaching strategies, continuation of the teaching sequences with possible teaching options. This approach using classroom situations was intended to evaluate the professional knowledge of teachers in a performance-oriented way as requested by Blömeke, Gustafsson and Shavelson (2014) summarising the discussion around competence assessments. Furthermore, the knowledge of students' error and its speedy recognition was tested with a time-limited test. In order to allow sound descriptions on the development of the professional knowledge partly shortened versions of the original TEDS-M tests on mathematics, mathematics pedagogy and general pedagogy were carried out transferred into a web-based design. 171 teachers from the secondary cohort and 130 teachers from the primary cohort participated once more in the study.

Professional knowledge of mathematics teachers – results of TEDS-FU

First results of the study FU on the development of early career teachers' professional knowledge reveal interesting insight into the structure and development of lower secondary teachers' professional knowledge.

The average level of MCK of these young teachers has decreased significantly between the first testing in 2008 at the end of their teacher education within TEDS-M and 2012 within TEDS-FU. By contrast, the average level of MPCK remained stable. The first result has been expected but the latter is more surprising as a decrease would have been plausible due to the nature of the paper-and-pencil test assessing partly declarative knowledge and measurement issues, i.e. the regression to the middle within repeated measurements and a positively selected sample. This result indicates the relevance of practical experience as learning opportunity for the development of MPCK, which is stated by the research on expertise for other professions already for a long time.

An analysis of the rank order of the participants regarding their achievements in MCK and MPCK in 2008 and 2012 yields interesting differences between MPCK and MCK: in MCK the rank order remains nearly unchanged, i.e. the knowledge level of the prospective teachers at the end of their education predicted very strongly the achievement level after four years of teaching practice. The situation concerning MPCK is varied: the level of MPCK at the end of teacher education predicts significantly the level of MPCK after four years of teaching, but the rank order of the mathematics teachers is less stable in this knowledge facet than in MCK. Referring to the research on expertise we can tentatively conclude that the MPCK of young teachers at the beginning of their career may be more flexible here. Teaching experience may be a strong opportunity to learn, influencing both knowledge facets. However, this influence may be much stronger concerning MPCK than towards MCK, which might be explained by differences in the nature of MPCK and MCK (see Buchholtz et al., 2014). In addition different ways in dealing with the experiences made in school practice might be relevant, a so-called "deliberate practice" can be important for the early career teachers' development but may vary inter-individually and by context.

Based on the TEDS-FU results, the relation between the knowledge facets and the young teachers' performance-oriented skills to perceive and interpret mathematics classroom situations analysed via path models cannot be described with a simple competence model, but require complex description. MCK and MPCK at the end of teacher education both predict significantly how well mathematics teachers can recognise time pressured student errors and how adequately they can notice the relevant activities in the classroom, interpret them and anticipate adequate options for further actions. However, the path model fits much better and explains more variance in the teachers' skills if the MCK and MPCK development between TEDS-M and TEDS-FU is taken into account (Blömeke et al., accepted).

The ability to notice classroom situations adequately and reason appropriately is influenced strongly by both knowledge facets, whereas the ability to recognise student errors depends more strongly on MCK than on MPCK. These results reveal once more the differences in the nature of MCK and MPCK (see Buchholtz et al., 2014).

Further evaluation of the TEDS-FU data on the nature of teacher expertise – describing the relation between knowledge, noticing and reasoning in classroom situations, and the speed of student error recognition – reveal unexpected results. If one distinguishes the facets of noticing and reasoning in classroom situations under an applied perspective, i.e. either content-related or pedagogical-oriented, the study points out that teacher expertise can neither be adequately described via models claiming either homogeneity of these indicators for expertise or by distinctions of facets according to domains or assessment methods. Based on our data, expertise can best be described with a two-dimensional model distinguishing between content-related knowledge (MCK, MPCK and speed in mathematics error recognition) and performance-related competencies (GPK, noticing and reasoning).

Analyses (based on IRT scaling and exploratory factor analysis) on GPK point out that the abilities to noticing and reasoning knowledge-based are in fact two loosely connected but different dimensions. The level of GPK at the end of teacher education does not predict these two abilities, which suggests that teachers' cognitions are reorganized during the transition into teaching. However, there exist relations between the current level of GPK and the ability to reasoning knowledge-based in contrast to noticing (for details see König et al., 2014).

Until now, it remains an open question as to whether teachers from primary levels have a similar structure of expertise, and if professional knowledge develops in the same manner or differently because of their different teaching practice. To summarise, the results of the studies described above show the differentiated nature of the expertise of mathematics teachers, the complicated interplay between the different facets of the professional knowledge of teachers and the high relevance of teaching practice for the development and the organisation of the professional knowledge of teachers in order to become true experts in their field..

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